The Modeling of Radar Image Based on Radar Textures

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Abstract. The paper is devoted to the development of a mathematical model of the terrain radar image created by the onboard radar. The model is based on the ray tracing method and takes into account the reflective properties of the surface using a special radar texture function The radar texture function is complex-valued function of the three 3D-vector arguments: a point of illuminated surface, a position of transmitter, a position of radiation receiver. This function describes amplitude and phase of reflected plane wave. The radar texture function is needed to simulate radar images formed in motion and to account for the reflective properties of various types of surfaces. The results of comparing the real radar image with the constructed model are also described.

INTRODUCTION

This paper is devoted to the development of a mathematical model of the terrain radar image (TRI) generated by the onboard radar. To implement the model, we use a ray tracing method based on the ideas of geometric and physical optics [1, 2, 3]. The reflective properties of the surface are take into account by using a special radar texture function [4] that describes amplitude and phase of reflected plane wave. We prefer to use this method because of its flexibility, which allows us to take into account wave re-reflections. In addition, it can be easily parallelized and, therefore, effectively implemented on modern supercomputers.

The paper describes computer software for the formation of radar images for a given digital terrain model. The software runs in parallel mode on supercomputers using MPI, OpenMP and allows to execute batch jobs parametrized by a 3D-scene file set. The main goal of our research is constructing the adequate mathematical TRI model that corresponds to a real radar image obtained for given conditions.

THE MATHEMATICAL MODEL OF TRI

The mathematical model of terrain radar image (TRI) is formed by summing complex-value amplitudes of signals reflected from surface of complicated scene. The method of geometric optic involves the ray tracing from antenna phase center (APC) to scene objects according to physical law of waves re-reflecting. The ray tracing is performed in the range of a given spatial angle Ω_φ corresponding to the scanning azimuth φ. Feature and novelty of the proposed model is the use of radar textures [4] to describe the reflective properties of surfaces. We will describe these textures in more detail below.

Consider the tracing process of an arbitrary ray with index m. This process starts from the point p_t of the phase center of the radar antenna. For the further convenience, we denote such point by $p_0 = p_t$. Further, denote as p_k^m the intersection point of this ray and of scene objects for the k-level of re-reflecting. Thus, p_1^m is point closest to p_0 where the ray intersects the scene. The point p_2^m is point closest to p_1^m on the ray re-reflected in accordance with geometric optical rules in point p_1^m, etc. We denote $r_{0,k}^m$ the distance from p_t (point p_0) to point p_k^m. Let $r_{k,l}^m$ denotes to the distance between points p_k^m and p_l^m.We also introduce the notion for the distance between p_t and the point of reflected signal having k-th re-reflecting level:

$$r_k^m = \frac{r_{0,k}^m + r_{k-1,k}^m + \ldots + r_{1,2}^m + r_{0,1}^m}{2} \tag{1}$$

In formula (1) we use notion $r_{k-1,k}^m$ for the distance between adjacent re-reflecting points while zero index corresponds to p_t point the location of the radar antenna phase center. The mathematical model of terrain radar image is described by the following equation:

$$I(r,\varphi) \approx \left| \sum_{\substack{ray(e_k^m)\in\Omega_\varphi \\ r-\frac{1}{2}\delta r \le r_k^m \le r+\frac{1}{2}\delta r \\ 1\le k\le K}} e_k^m(r_k^m, p_t, \varphi) \right|, \qquad (2)$$

where $I(r,\varphi)$ denotes the model of radar image as a function of distance r and azimuth φ, $ray(e_k^m)$ is a direction of k-th level of re-reflecting of m-th partial signal, Ω_φ is a spatial angle of the antenna diagram of azimuthal direction φ, δr is a given radar resolution by distance, K is maximal level of re-reflection, $|\cdot|$ is a module of a complex-valued number.

The quantity $e_k^m(r_k^m, p_t, \varphi)$ is a complex-valued amplitude of k-th re-reflection level of m-th partial signal incoming to point p_t from distance r_k^m and from the φ azimuth direction :

$$e_k^m(r_k^m, p_t, \varphi) = Q_k \cdot \frac{\sqrt{G_\varphi(p_k^m)}}{r_{o,k}^m} \cdot e^{-j\frac{2\pi}{\lambda}r_k^m} \cdot \frac{\dot{T}(p_k^m; p_0, p_{k-1}^m)}{r_{k-1,k}^m} \cdot \prod_{z=1}^{k-1} \frac{\dot{T}(p_z^m; p_{z+1}^m, p_{z-1}^m)}{r_{z-1,z}^m} \cdot \sqrt{G_\varphi(p_1^m)}.$$

Here,

$$Q_k = \frac{1}{M}\frac{Q_{per}}{(4\pi)^{k+1}},$$

where Q_{per} is a normalized transmission power of radar, M is a total number of traced rays, $G_\varphi(p)$ is a function of the antenna diagram having an axis oriented by an azimuth φ, and function value corresponds to direction from p_t to p, $\dot{T}(p; p_t, r_r)$ is a complex-valued function of radar texture containing the information about reflecting surface properties, where p is a point of texture, p_t is a point of radiation transmitter, p_r is a point of radiation receiver, j is an imaginary unit, $\frac{2\pi}{\lambda}r_k^m$ is a phase of m-th partial signal received from distance r_k^m, and λ is a wave length.

The mathematical model of radar image implemented in our software is matrix $I(r,\varphi)$. In fact, this matrix is a sampled function of the amplitude of the trajectory radio signal reflected by the scene depending on distance r to reflecting point and azimuth φ. The radar texture in model 2 has the same meaning as the specific radar cross-section (SRCS) and scattering indicatrix applying for the description of the reflective surface properties. This new term is necessary for emphasizing explicit dependence of modeled reflective surface properties on the observer location. The radar texture is defined by the following formula:

$$\dot{T}(p_i; p_t, p_r) = T(p_i; p_t, p_r)e^{j\psi_i},$$

where p_i is a point of texture, p_t is a point of radiation source, p_r is a point of radiation receiver, $T(\cdot)$ is an amplitude, ψ_i is a phase of re-reflected plane wave in p_i, j is an imaginary unit. If the outcoming ray from the origin of spherical coordinate system to direction determined by angles (φ,θ) intersects scene surface in the point that is closest to the origin and is situated at the distance r, then we set $\dot{T}(r,\varphi,\theta)$ value as complex-valued reflection coefficient in this point of scene surface. For the remaining points of the given ray we set $\dot{T}(r,\varphi,\theta) = 0$. If the considered ray does not intersect scene surface, then we set $\dot{T}(r,\varphi,\theta) = 0$ for all points belonging to this ray.

The functions $\dot{T}(r,\varphi,\theta)$ and $\dot{T}(p_i; p_t; p_r)$ are correlated evidently: $p_t = p_r$; $r = |p_i - p_t|$; φ,θ) are orientation angles of the vector $p_i - p_t$ that determine an angle of incidence ν of a wave in the point p_i from the point p_t. With equality $p_t = p_r$ the module of radar texture is defined by the following formula:

$$T(p_i; p_t; p_t) = \sqrt{\sigma_0(\nu_i(p_i, p_t))},$$

where $\sigma_0(\nu_i(p_i, p_t))$ is SRCS of surface in point p_i directed to p_t, $\nu_i(p_i, p_t)$ is a corresponding angle of incidence in p_i from p_t.

The values of SRCS are taken from the database of reflective surface properties (DB RSP). The content of this database was made from sources [5, 6]. The information in these sources has a view as experimentally derived graph dependencies of various physical values describing reflective surface properties. These graphs are dependencies of SRCS on angle of incidence ν for various surface types. When the data is absent in DB RSP we use the diffuse-reflected model of a radar texture:

$$T(p_i; p_t; p_r) = K_d \cos\nu + (1-K_d)(\cos\alpha)^\rho.$$

Here $K_d \geq 0$ is a coefficient of diffuse (Lambert) scattering, α is an angle between the direction of reflection and the direction to an observer, reflection occurs by geometric optics principle , $\rho > 0$ is an indicator of specular reflection.

The random character of reflection of electromagnetic waves in the TRI model is taken in account using the following formula:

$$\dot{T}(p,\varphi,\theta) = \overline{T(\rho,\varphi,\theta)}\varepsilon(q(\rho,\varphi,\theta))e^{j\psi}, \tag{3}$$

where $\varepsilon(q)$ is a two-dimensional ($q \in R^2$) real given stochastic process normalized to an average amplitude $\overline{T(\rho,\varphi,\theta)}$, ψ is an random phase uniformly distributed in $[0,2\pi]$, $q(\rho,\varphi,\theta)$ is an intersection point of ray (ρ,φ,θ) with surface given in the local system coordinates of plane primitive. This random phase is the same for the all rays intersecting the same primitive that has a triangular form.

The noises of receiver are modeled by additive not-correlated normal noise that is a random value N having zero average and some variance σ_N^2. The noise N is added to $I(r,\varphi)$.

SIMULATION OF A RANDOM TWO-DIMENSIONAL FIELD

The value of reflected signal depends on many factors some of which have a random nature. These factors are difficult to account for in TRI mathematical model. Therefore, it is necessary to provide a way of randomness generation.

Consider examples of random factors:

- roughness of relief;
- the value of incidence angle ν;
- the complex-valued dielectric constant of reflective surface;
- the carrier frequency (the wave length λ);
- the polarization of transmitted and received electrical field (VV, HH etc.);
- the attenuation and scattering of radio waves.

To simulate random reflection fluctuations we use two-dimensional random field distributed by Rayleigh and with normal correlation function. For this purpose we use algorithm described in [7]. For the normalized Rayleigh distribution [5] with density function $p(u)$ we set σ to $\sqrt{\frac{2}{\pi}}$. Then, the distribution characteristics take form:

$$\begin{gathered} p(u) = \frac{\pi u}{2} e^{-\frac{\pi u^2}{4}}, u \geq 0, \\ m_u = 1, \\ \sigma_u^2 = \left(\frac{4}{\pi} - 1\right), \end{gathered} \tag{4}$$

where m_u is an expectation, σ_u^2 is a variance.

The correlation function for non-centered processes with distribution of Rayleigh (such that $R(0) = m_R^2 + \sigma_R^2 = 2\sigma^2$, where σ is a distribution parameter) has the following form [8]

$$R(\rho) = m_R^2 + \sigma_R^2 \cdot r(\rho), \tag{5}$$

where m_R^2 is a square of an expectation of random value distributed of Rayleigh, σ_R^2 is a variance of random value, $r(\rho)$ is a normalized correlation function of centered process.

The normal correlation function with consideration formula (5) and characteristics (4) of normalized Rayleigh distribution has the form:

$$R(\rho) = 1 + \left(\frac{4}{\pi} - 1\right) e^{-\beta^2\rho^2}. \tag{6}$$

Therefore, the given problem is reduced to modeling of random field with correlation function (6) and distribution having characteristics (4). The random value Z having Rayleigh distribution can be calculated by formula from [9]

$$Z = \sqrt{X^2 + Y^2}, \tag{7}$$

where X, Y are random independent normal values with zero averages and equal variances. Formula (7) is also applied for random field modeling with Rayleigh distribution, where X, Y are random normal fields with zero averages and equal variances.

Thus, random two-dimensional field simulation algorithm is as follows:

1. We obtain the normalized correlation function from (5) and (6):

$$r(\rho) = e^{-\beta^2 \rho^2} \tag{8}$$

2. By non-linear transformation of given correlation function $r(\rho)$ for Rayleigh distribution case [8, 9] we obtain normalized correlation function $r_0(\rho)$:

$$r_0(\rho) \approx \sqrt{r(p)} = e^{-\frac{\beta^2}{2}\rho^2} \tag{9}$$

3. We model two random normal fields ζ_1 and ζ_2 with zero averages, unit variances, and normalized correlation function (9).

   3.1. The normal two-dimensional white noise with zero average and unit variance is modeled. Then, we store the resulting field as matrix in variable $F_{m,n}$.

   3.2. We obtain forming function by normalized correlation function (9) (the algorithm of the calculation of such function is contained in [7]):

$$f(\rho) = e^{-\beta^2 \rho^2}.$$

   The matrix entries of the forming function are stored in $f_{p,r}$ under the condition that $\rho = \sqrt{p^2 + r^2}$.

   3.3. We compute amplitudes of normal field with normalized correlation function $r_0(\rho)$ as convolution of white noise and forming function by formula:

$$U_{m,n} = \sum_{p=-p_0}^{p_0} \sum_{r=-r_0}^{r_0} F_{m-p,n-r} f_{p,r}.$$

   In the same way, we form $\zeta_2 \sim N(0,1)$ with correlative function $r_0(\rho)$ (9).

4. The amplitudes of the desired random field are computed by formula from [9]

$$\varepsilon(m,n) = \sigma \sqrt{\zeta_1^2[x,y] + \zeta_2^2[x,y]},$$

   where $\varepsilon(m,n)$ is a discrete representation of two-dimensional stochastic process $\varepsilon(q)$ (from (3)), $\sigma = \sqrt{\frac{2}{\pi}}$ is a distribution parameter.

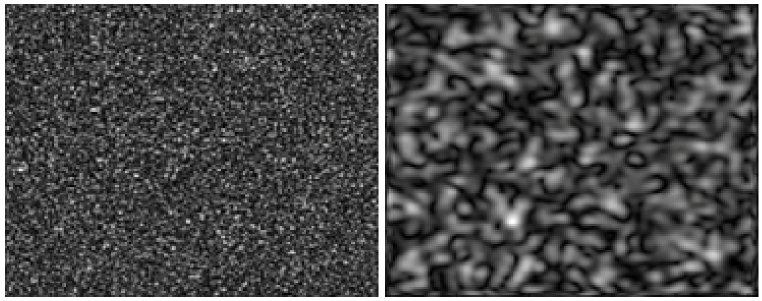
We compute the correlation radius for normalized correlation function $r(\rho) = e^{-\beta^2 \rho^2}$. We choose $r(\rho) = 0.5$, and then we have:

$$e^{-\beta^2 \rho^2} = \frac{1}{2}$$

Hence, we obtain the formula for correlation radius in discrete values:

$$\rho(\beta) = \frac{\sqrt{\ln 2}}{\beta} \approx \frac{0.83}{\beta}$$

Figure 1 illustrates the results of fields simulation with Rayleigh distribution whose correlation radii are $\rho = 1$ and $\rho = 5$ respectively, and with normal correlation function (6). The fields size is 256×256 pixels.



(a) (b)

FIGURE 1: The fields with Rayleigh distribution, correlation function (6), and correlation radii (a) $\rho = 1$ and (b) $\rho = 5$

TERRAIN RADAR IMAGES SIMULATION (TRIS) SOFTWARE

We use previously developed simulation software [10] for forming terrain radar images (TRIS sowtware) by a given raster digital model of surface (RDMS). This software consists of:

- the program 'TRIS-configurator' that formed the computational task files formatted as json;
- the program 'TRIS- calculator' is the main computational module formed the model of radar image;
- - the program 'TRIS-viewer' is to view TRI.

The raster digital model of surface (RDMS) consists of multilevel radar raster map and geometric 3D-scene with imposed radar texture. The source data for forming RDMS includes the follows:

- the data of the remote sensing of Earth consisting from high resolution panchromatic stereopairs and from multispectral image;
- the mathematical models of reflective surface properties;
- the data from DB RSP.

THE SIMULATION RESULTS

We consider two computational experiments as examples of TRIS software working.

Experiment 1

We place here the computational experiment for scene TRI modeling. The fragment of 3D-scene model can be seen at Figure 2. This scene is a terrain piece of size 4751×4999 m. The size of scene by OZ axis is from 112 to 235 m. Figure 3 illustrates the TRI model corresponding 3D-scene the piece of that is shown at Fig. 2. This radar image is obtained in scanning by real ray mode for the following parameters:

- the angle size of antenna pattern by $\varphi = 0.1^o$;
- the angle size of antenna pattern by $\theta = 1^o$;
- the wave length $\lambda = 3mm$;
- the scan range = 60^o;
- the resolution $\delta r = 1m$;
- the sector of ray tracing $\Omega = [-45^o, +45^o] \times [-45^o, +45^o]$.

FIGURE 2: The piece of 3D-scene model

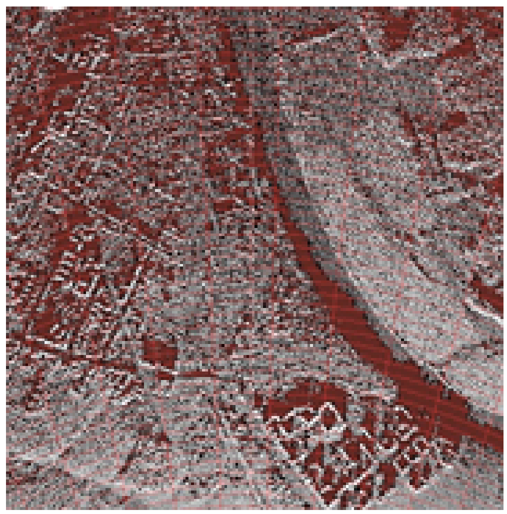


FIGURE 3: The model of terrain radar image

Experiment 2

Consider the second experiment that concerns the comparing model and real radar images of city areas (Fig. 4). The real radar image (Fig. 8) is obtained by the radar 'Locia' with parameters as shown in Table 1. In this experiment

TABLE I: The parameters of radar 'Locia'

Description	Characteristics
Antenna gain	$K_y = 288$
Wave length	$\lambda = 3.2cm$
Band width	$\delta f = 20MHz$
Peak pulse power	$P_H = 0.3Wt$
The width of antenna pattern by azimuth	$\delta\varphi = 1.7^o$
The width of antenna pattern by position angle	$\theta = 18^o$

the 3D-scene model (Fig. 5) includes only those houses that can be seen on real image. This was made because some houses are overlapped by plants since the scanning was made from the roof building. Currently, we do not take in account plants in our model and plan to realize it in future researching. Figures 6 and 7 show the model radar image of the city area and combination of the real and model images.

FIGURE 4: The map of city area. The red circle corresponds to the radar location while the red circuit illustrates scan range. The houses are included in 3D-scene are marked black.

FIGURE 5: 3D-scene model

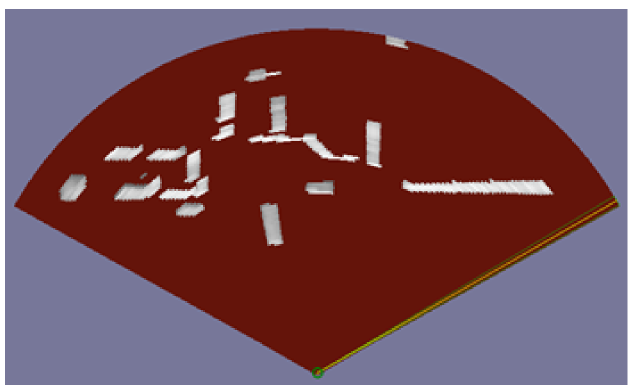


FIGURE 6: The model radar image is constructed by the 3D-scene from Fig.5 with parameters of TRI as from Table 1

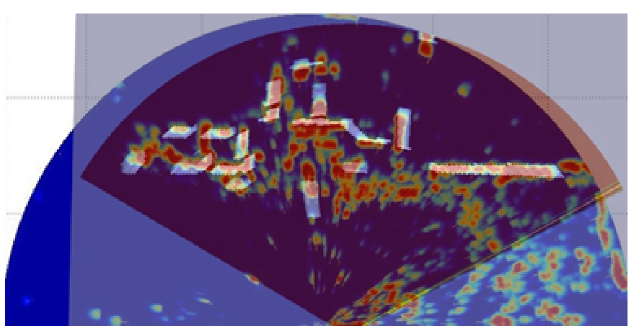


FIGURE 7: Combination of the real and model radar images

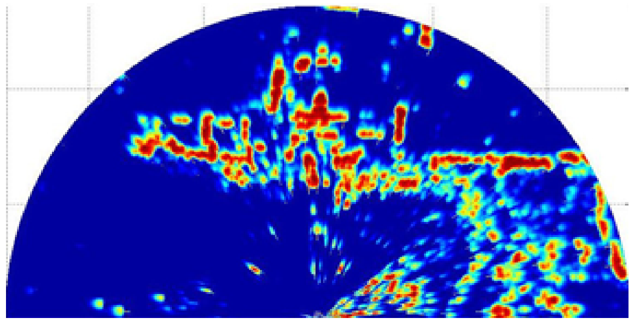


FIGURE 8: The real image obtained by radar 'Locia'

CONCLUSION

We developed the mathematical model of terrain radar image based on the geometric optics method using proposed radar texture method. The experiments carried out showed, that:

- The characteristics of formed radar images are corresponded to the theoretical models of wave scattering for various types of surfaces.
- We implemented the simulation of radar image in the mode of real aperture in various meteorological conditions.
- The generated trajectory signal corresponds to the given radar characteristics and vehicle moving parameters.

In the following, we plan to realize criterions of comparing radar images and carried out research of correctness of model radar image comparing with real radar image obtained by observation known scene and with known conditions.

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