Mathematical Modeling and Optimization of Operation of the Nuclear Fuel Reloading Mechanism for the BN-800 Reactor

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Abstract. For the BN-800 reactor, for which the refueling mechanism consists of three rotating plugs, a time-optimal algorithm is proposed that works on the assumption that only two plugs can rotate at the same time. Application of such an algorithm will help to reduce the shutdown time of the power unit for refueling and, accordingly, increase the utilization factor of the installed capacity of the NPP unit.

INTRODUCTION

A promising large-scale nuclear power industry should have guaranteed safety, economic stability and competitiveness, the absence of restrictions on the resource base for a long period of time, and environmental sustainability (low waste). These conditions are met by nuclear power systems with fast neutron reactors (RBR) with a liquid metal coolant [1]. Russia has many years of experience in the construction and operation of sodium-cooled fast reactors. The BN-600 fast neutron reactor has been reliably and safely operated at Beloyarskaia NPP for 40 years, and the new BN-800 reactor has already been operating for 5 years. The power density in the core of fast neutron reactors (RBR) is about 500 MW/m3, which is 50 times higher than in RBMK reactors and 5 times higher than in WWER reactors. Sodium is used to ensure reliable heat dissipation. Being a highly efficient heat carrier, sodium has a number of disadvantages, first of all it is high chemical activity with respect to water and air. The presence of sodium in the reactor vessel requires its reliable isolation from the environment in all operating modes, including refueling. Therefore, the rotation of eccentrically located rotary plugs with hydraulic seals filled with tin-bismuth alloy [2] is used to guide the reloading mechanism to the fuel assembly to be reloaded. On other types of reactors, the reloading machine is guided along two mutually perpendicular coordinates. To improve the efficiency of a nuclear power plant, it is necessary to ensure maximum power generation, i.e. to minimize downtime for repairs and refueling.

To unload a burned-out fuel assembly (FA) from the reactor, the gripper of the refueling machine must move to its coordinates, lower the gripper, perform engagement, lift the gripper from the fuel assembly, transfer it to the fuel pool, lower it, release and raise the gripper. These operations are performed for each unloaded fuel assembly, regardless of the reloading order. In the reverse order, fresh fuel assemblies are loaded. The time spent on the operation of lowering-lifting the gripper of the reloading machine, coupling-uncoupling the gripper from the fuel assembly can be ignored in the calculation algorithm, since it is a parameter that cannot be optimized in this context. As a result, the problem can be represented as flat. The problem of optimizing the sequence of operations for rearranging fuel assemblies is to minimize the time for replacing nuclear fuel and, accordingly, reduce the downtime of a nuclear power unit.

The mechanism of guidance in fast neutron reactors to capture the required coordinates of the fuel assembly is carried out by rotating eccentrically located devices (rotary plugs). The refueling system of the BN-800 reactor is intended for refueling the fuel assemblies and consists of a set of units that ensure the guidance of the refueling mechanism to the specified coordinates, capture, raise, lower and rotate the assemblies. On the neck of the BN-800 reactor vessel there are three rotary plugs, the smaller of them is located inside the middle and the middle inside

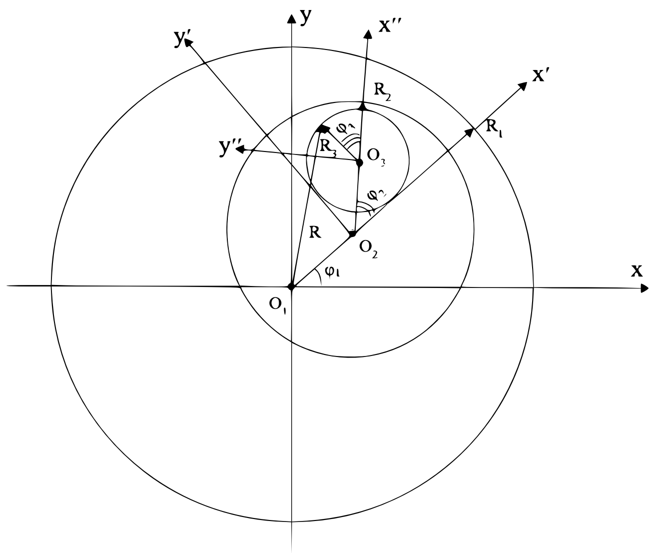

FIGURE 1. Scheme of rotary plugs.

the large plug. The smaller plug contains the fuel assembly gripping mechanism. The plugs, which are the lid of the reactor, also serve as thermal and biological protection. In this paper, we will consider the problem of the fastest guidance of a gripper located on a smaller plug to a given fuel assembly under the assumption that no more than two plugs can simultaneously rotate at each moment. The solution to this problem will help to reduce the shutdown time of the power unit for refueling operations. A mathematical model has been built that describes the movement of three connected plugs. On its basis, an algorithm for constructing an optimal control is proposed under the assumptions made.

MATHEMATICAL MODEL DESCRIBING THE PLUGS DYNAMICS

The mechanical system consists of three turning plugs (see Fig. 1). A large plug is a disk of radius R_1, the geometric center of which remains motionless during movement. The large plug has an eccentric circular notch of radius R_2, inside of which an average plug is placed, which is a disk of radius R_2, the geometrical center of O_2 of which remains motionless while moving relative to a large plug. The middle plug has an eccentric circular cutout of radius R_3. Inside this cutout there is a small plug, which is a disk of radius R_3, the geometric center O_3 of which, while moving, remains stationary relative to the average cork. We will neglect the consideration of friction forces in the interaction of plugs. A mathematical model of the overload mechanism under the assumptions made is obtained in [3] in the form of Lagrange equations of the second kind. As generalized coordinates, we choose the angle of rotation of the large plug — φ_1, the angle of rotation of the middle plug — φ_2 with respect to the large plug and the angle of rotation of the small plug — φ_3 with respect to the middle plug. The generalized forces are the control moments u_1, u_2, and u_3 applied to the large, medium and small plugs, respectively.

Let us denote by J_1, J_2 and J_3 are the moments of inertia of the large, medium plugs with notches and the small plugs relative to the O_1, O_1, and O_1, axes, respectively, m_2 is the mass of the average plug with a notch, m_3 is the mass of the small plug, $e_2 = |O_1O_2|$, $e_3 = |O_2O_3|$, $a = |O_2C_2|$, $\alpha = \angle(O_3O_2C_2)$, C_2 is the center of mass of the middle plug with a notch, and O_3 is the center of mass of the small plug.

The mathematical model of the transfer device according to [3] is described using the following Lagrange equations of the second kind:

$$J_1\ddot{\varphi}_1 + J_2(\ddot{\varphi}_1 + \ddot{\varphi}_2) + J_3(\ddot{\varphi}_1 + \ddot{\varphi}_2 + \ddot{\varphi}_3) + m_2\left(e_2^2\ddot{\varphi}_1 + e_2 a\cos(\varphi_2+\alpha)(2\ddot{\varphi}_1 + \ddot{\varphi}_2) - e_2 a\sin(\varphi_2+\alpha)(2\dot{\varphi}_1 + \dot{\varphi}_2)\dot{\varphi}_2\right)$$

$$+m_3\left(e_2^2\ddot{\varphi}_1 + e_3^2(\ddot{\varphi}_1 + \ddot{\varphi}_2) + e_2e_3\cos(\varphi_2)(2\ddot{\varphi}_1 + \ddot{\varphi}_2)\ - e_2e_3\sin(\varphi_2)(2\dot{\varphi}_1 + \dot{\varphi}_2)\dot{\varphi}_2\right) = u_1,$$

$$J_2(\ddot{\varphi}_1 + \ddot{\varphi}_2) + J_3(\ddot{\varphi}_1 + \ddot{\varphi}_2 + \ddot{\varphi}_3) + m_2e_2a\left(\cos(\varphi_2+\alpha)\ddot{\varphi}_1 + \sin(\varphi_2+\alpha)\dot{\varphi}_1^2\right)$$

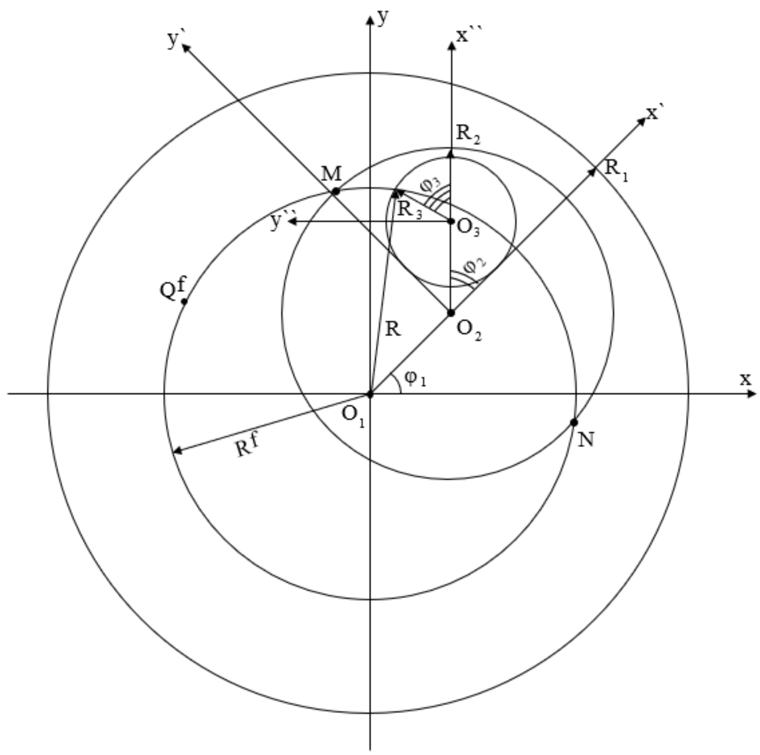

FIGURE 2. Arc MN and final point Q^f

$$+m_3\left(e_3^2(\ddot{\varphi}_1+\ddot{\varphi}_2)+e_2e_3(\cos(\varphi_2)\ddot{\varphi}_1+\sin(\varphi_2)\dot{\varphi}_1^2)\right)=u_2,$$

$$J_3\left(\ddot{\varphi}_1+\ddot{\varphi}_2+\ddot{\varphi}_3\right)=u_3. \tag{1}$$

The problem of optimizing the sequence of operations for rearranging fuel assemblies (FA), control and protection rods (CPS) during refueling of the reactor is to minimize the refueling time of the reactor and, accordingly, reduce the downtime of the NPP power unit [2]. The considered optimization problem has two components: it is a routing problem that determines the order of rearrangement of fuel assemblies and the task of optimizing the operation of the overload mechanism when moving one fuel assembly. The routing problem was considered under some assumptions in [4]. In this work, the route component of the problem is not considered.

Note that the mathematical model of the overload mechanism (1) is essentially non-linear. In [3] we considered the case when the rotary plugs rotate sequentially. To optimize the rotation of one plug, the results obtained in [5, 6] were used. As a result, the problem was reduced to a nonlinear programming problem with three variables. In this paper, we consider the case when no more than two plugs can rotate simultaneously. The technological capabilities of the BN-800 reloading device make it possible to perform such operations.

We will assume that first a simultaneous reversal of the middle and small plugs is carried out, and then the large plug unfolds. Obviously, in this case, with the simultaneous turning of the middle and small plugs, the gripper of the transfer device should be on a circle with the center coinciding with the center of the large plug (see Fig. 2). In Figure 2, the point Q_f is the point to which the gripper of the transfer device should be moved as a result.

OPTIMIZATION PROBLEM

Let us construct a mathematical model describing the movement of the middle and small plugs under the assumption that the large plug is stationary (i.e. $\varphi_1(t) = const$. It follows from (1) that the dynamics of the middle and small plugs in this case will be described by the system of equations

$$\ddot{\varphi}_2=\bar{a}_1(u_2-u_3),$$

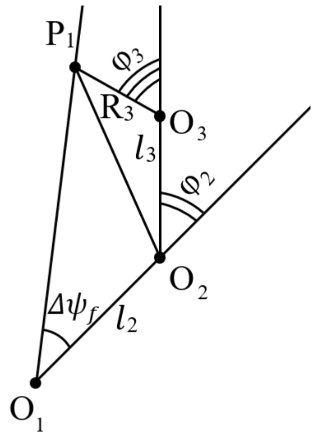

FIGURE 3. Points P_1 and P_2

$$\ddot{\varphi}_3 = -\bar{a}_1 u_2 + \bar{a}_2 u_3. \tag{2}$$

Here

$$\bar{a}_1 = \frac{1}{J_2 + m_3 e_3^2}, \ \ \bar{a}_2 = \frac{J_2 + J_3 + m_3 e_3^2}{J_3(J_2 + m_3 e_3^2)}.$$

The end point for a joint reversal of the middle and small traffic jams is designated by P_1. Note that the point P_1 is determined by the values of the angles φ_1^0, φ_2^f, φ_3^f). According to [5, 6], if the inequality

$$\bar{a}_2|\Delta\chi| \geq \frac{\mu_1}{\mu_2}\bar{a}_1|\Delta\varphi|, \tag{3}$$

where

$$\varphi = \varphi_2 + \varphi_3, \ \Delta\varphi = \varphi_2^f + \varphi_3^f - \varphi_2^0 - \varphi_3^0, \ \chi = \varphi_2 + \frac{\bar{a}_1}{\bar{a}_2}\varphi_3, \ \Delta\chi = \varphi_2^f + \frac{\bar{a}_1}{\bar{a}_2}\varphi_3^f - \varphi_2^0 - \frac{\bar{a}_1}{\bar{a}_2}\varphi_3^0, \tag{4}$$

φ_2^0, φ_3^0 is an initial values of the angles φ_2, φ_3, φ_2^f, φ_3^f final values of the angles φ_2, φ_3 with a joint turn of the middle and small plugs. The optimal turnaround time in this case according to [5, 6] is determined by the formula

$$\vartheta_1 = 2\sqrt{\frac{\bar{a}_2|\chi(\vartheta_1) - \chi(0)|}{\bar{a}_1\mu_1(\bar{a}_2 - \bar{a}_1)}}. \tag{5}$$

and

$$u_2(t) = \mu_1 \operatorname{sign}(\chi(\vartheta_1) - \chi(0)) \operatorname{sign}\left(\frac{\vartheta_1}{2} - t\right), \tag{6}$$

$$u_3 = \beta_1 \operatorname{sign}\left(t - \frac{\vartheta_1}{2}\right) = \frac{4\Delta\varphi}{(\bar{a}_2 - \bar{a}_1)\vartheta_1^2}\operatorname{sign}\left(\frac{\vartheta_1}{2} - t\right). \tag{7}$$

If the inequality

$$\bar{a}_2|\Delta\chi| \leq \frac{\mu_1}{\mu_2}\bar{a}_1|\Delta\varphi|, \tag{8}$$

holds, then the optimal turnaround time for medium and small traffic jams is determined by the formula

$$\vartheta_2 = 2\sqrt{\frac{|\varphi(\vartheta) - \varphi(0)|}{\mu_2(\bar{a}_2 - \bar{a}_1)}}. \tag{9}$$

In this case, the optimal control takes the form

$$u_2 = \frac{4\bar{a}_2(\Delta\varphi_2 + \frac{\bar{a}_1}{\bar{a}_2}\Delta\varphi_3)}{\bar{a}_1(\bar{a}_2 - \bar{a}_1)\vartheta_2^2}\mathrm{sign}\left(t - \frac{\vartheta}{2}\right), \tag{10}$$

$$u_3(t) = \mu_2 \,\mathrm{sign}\,(\varphi(\vartheta) - \varphi(0))\ \mathrm{sign}\left(\frac{\vartheta}{2} - t\right). \tag{11}$$

The point P_1 is located on the arc MN (see Fig. 2), the angles φ_2 and φ_3 do not exceed π in absolute value. Denote the angle $\angle O_2O_1P_1 = \Delta\psi_f$. Note that the point P_1 can be specified by a set of angles $(\varphi_1^0, \varphi_2^f, \varphi_3^f)$ or by an angle

$$\psi_f = \varphi_1^0 + \Delta\psi_f, \tag{12}$$

which is the segment 0_1P_1 with the OX axis. We will assume that the angle ψ_f is given. We will show how to define the angles $\varphi_2^f, \varphi_3^f)$ using the angle ψ_f.

Consider a triangle $\triangle\ O_1O_2P_1$. According to the cosine theorem

$$|O_2P_1|^2 = R_f^2 + e_2^2 - 2R_fe_2\cos\Delta\psi_f \tag{13}$$

or from the formula (see Fig. 2)

$$|O_2P_1|^2 = (R_f\cos\psi_f - e_2\cos\varphi_1^0)^2 + (R_f\sin\psi_f - e_2\sin\varphi_1^0)^2. \tag{14}$$

From the triangle $\triangle\ O_1O_2P_1$ it follows that

$$\angle P_1O_2O_1 = \arccos\frac{R_f^2 - e_2^2 - |O_2P_1|^2}{2e_2|O_2P_1|}. \tag{15}$$

From the triangle $\triangle\ O_2O_3P_1$ it follows that

$$\angle O_3O_2P_1 = \arccos\frac{R_3^2 - e_3^2 - |O_2P_1|^2}{2e_3|O_2P_1|} \tag{16}$$

and

$$\angle P_1O_3O_2 = \arccos\frac{|O_2P_1|^2 - R_3^2 - e_3^2}{2e_3R_3}. \tag{17}$$

Therefore

$$\varphi_2^f = \pi - \angle P_1O_2O_1 - \angle O_3O_2P_1; \quad \varphi_3^f = \pi - \angle O_2O_3P_1. \tag{18}$$

After, as a result of the joint reversal of medium and small traffic jams, the gripper of the transfer device will be transferred to the point P_1. Further movement of the gripper from the point P_1 to the point Q^f will be carried out by reversing the large congestion.

The second option of moving from the starting point to the point Q^f is also possible. First, as a result of the joint reversal of the middle and small traffic jams, the grip moves to the point P_2 (see Fig. 2, 3), and then from the point P_2 along the arc of a circle of radius R^f to the point Q^f. Fig. 3 shows that

$$|O_1O_3|^2 = e_2^2 + e_3^2 - 2e_2e_3\cos(\pi - \varphi_2^f). \tag{19}$$

Also from fig. 3 we find that

$$\angle LO_3P_1 = \angle P_2O_3L = \varphi_3^f + \beta$$

and

$$\beta = \arccos\frac{e_2^2 - e_3^2 - |O_1O_3|^2}{2|O_1O_3|e_3}.$$

Let's introduce the notation: $\angle LO_3P_2 = \varphi_3^{f-}$. Then

$$\varphi_3^{f-} = -2\beta - \varphi_3^f.$$

Note that the point P_2 is specified by the angles $\varphi_1^0, \varphi_2^f, \varphi_3^{f-}$. Knowing the angles for the start and end points, similarly to (5) and (**??**) it is fashionable to find the time required to move from the starting point to the point P_2 (in the formulas (5) and (23) instead of φ_3^f, you need to substitute φ_3^{f-}).

Let us write down the equations of the dynamics of the system in this case. Assuming in the system (1) $\varphi_2(t) = \varphi_3(t) = const$, we obtain that the angle $\varphi_1(t)$ will satisfy the equation

$$J_1^* \ddot{\varphi}_1 = u_1, \tag{20}$$

where

$$J_1^* = J_1 + J_2 + J_3 + m_2 e_2^2 + 2e_2 a \cos(\varphi_2^f + \alpha) + m_3(e_2^2 + e_3^2 + 2e_2 e_3 \cos(\varphi_2^f)).$$

According to [3], the optimal travel time from a point on the arc MN to the point Q^f is

$$\vartheta_3 = 2\sqrt{\frac{J_1^*}{\mu_1}|\Delta\varphi_1|}, \tag{21}$$

where $\Delta\varphi_1 = \angle P_1 O_1 Q^f$ is the angle to which the gripper will move from P_1 along the arc to Q^f, or $\Delta\varphi_1 = \angle P_2 O_1 Q^f$ if the gripper will move to Q^f from P_2.

The total time required for such a movement is determined by the formula

$$T = \vartheta_1 + \vartheta_3, \tag{22}$$

if holds the condition (3), and

$$T = \vartheta_2 + \vartheta_3, \tag{23}$$

if holds the condition (8). We will calculate the travel time from the starting point to the P_1 point and then to the Q^f point. We will also calculate the travel time from the starting point to the P_2 point and then to the Q^f point. We choose the smallest of the two times obtained.

Now let's determine the coordinates of the points M and N. To the point M we assign the angle γ_1, which is counted from the OX axis in the counterclockwise direction to the point M. Similarly, point N is associated with the angle γ_2, which is counted from the OX axis in the counterclockwise direction to the point N. Then the coordinates of the points M and N will have the form $X = R_f \cos\gamma_i,\ Y = R_f \sin\gamma_i\ (i = 1, 2)$. The equation of the circle of radius R_2 centered at the point O_2 has the form:

$$(X - e_2 \cos\varphi_1^0)^2 + (Y - e_2 \sin\varphi_1^0)^2 = R_2^2.$$

Substituting into it the previously written expressions for X and Y, we obtain an equation from which the angles are determined

$$\gamma_1 = \varphi_1^0 + \arccos\frac{R_f^2 + e_2^2 - R_2^2}{2e_2 R_f};\ \gamma_2 = \varphi_1^0 - \arccos\frac{R_f^2 + e_2^2 - R_2^2}{2e_2 R_f}. \tag{24}$$

Next, we discretize the arc MN and calculate the best time by going through the points on this arc and looking for the minimum value.

In the first case, the starting point is specified by the conditions $\varphi_1^0 = 80^{circ}$, $\varphi_2^0 = 10^{circ}$, $\varphi_3^0 = 10^{circ}$. The final point of Q^f has coordinates $(0.99; -0.06)$. In fig. 4 shows a graph in blue when switching from the mode of joint rotation of the second and third plugs to the mode of rotation of the first plug occurs at the point P_1 (the abscissa shows the possible position of the point P_1 on the arc MN), the graph is shown in brown when the first mode changes to the second at the point P_2. Fig. 4 that the shortest time to complete the movement from the starting point to the ending point will be in the case when the point P_1 coincides with the point M.

The following graph shows the simulation results for the case when the starting point has coordinates $\varphi_1^0 = 10°$, $\varphi_2^0 = 20°$, $\varphi_3^0 = 20°$. The final point of Q^f has coordinates $(0.97; -0.21)$. In fig. 5, as in the previous case, the blue color is a graph illustrating the case when the mode switching occurs at the point P_1, and the brown color is the graph when the mode switching occurs at the point P_2. The graphs show that in this case the optimal result is achieved when switching occurs at the point P_2.

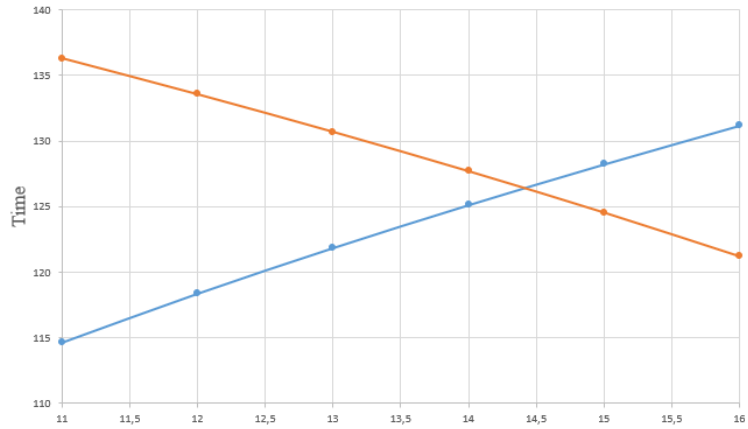

FIGURE 4. Results of the first computational experiment

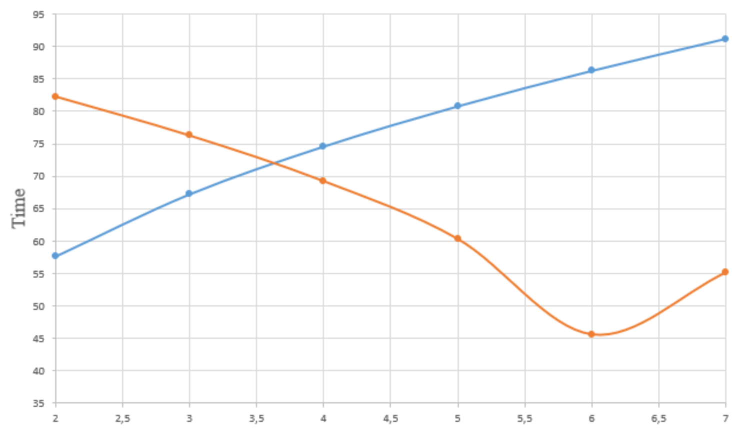

FIGURE 5. Results of the second computational experiment

CONCLUSIONS

The article proposes a mathematical model and considers the problem of controlling the mechanism for reloading nuclear fuel for the BN-800 reactor. The peculiarity of the overload mechanism is that the mechanism for aiming at the fuel assembly consists of three cylinders inserted one into the other with offset centers. Under some assumptions, an algorithm is proposed that provides guidance for the capture of the refueling mechanism in the shortest time. The proposed algorithm will help to reduce the time required for refueling and, accordingly, will reduce the shutdown time of the NPP power unit.

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