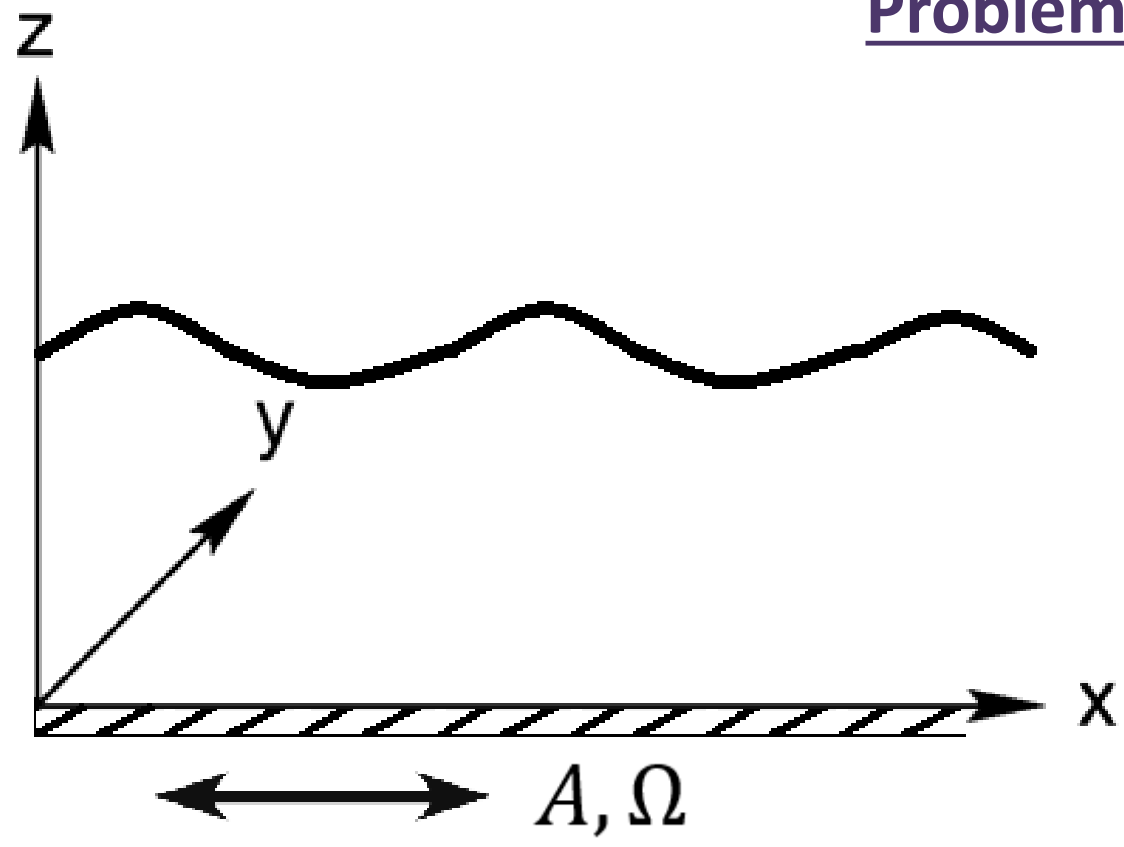


INFLUENCE OF LONGITUDINAL VIBRATIONS OF THE PULSATION FIELDS OF LONG-WAVE MARANGONI CONVECTION IN THIN FILM

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Problem formulation



$$\begin{aligned} \operatorname{div} \mathbf{v} &= 0 \\ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \nabla \mathbf{v} &= -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{v} - g \mathbf{k} \\ \frac{\partial T}{\partial t} + \mathbf{v} \nabla T &= \chi \Delta T \end{aligned}$$

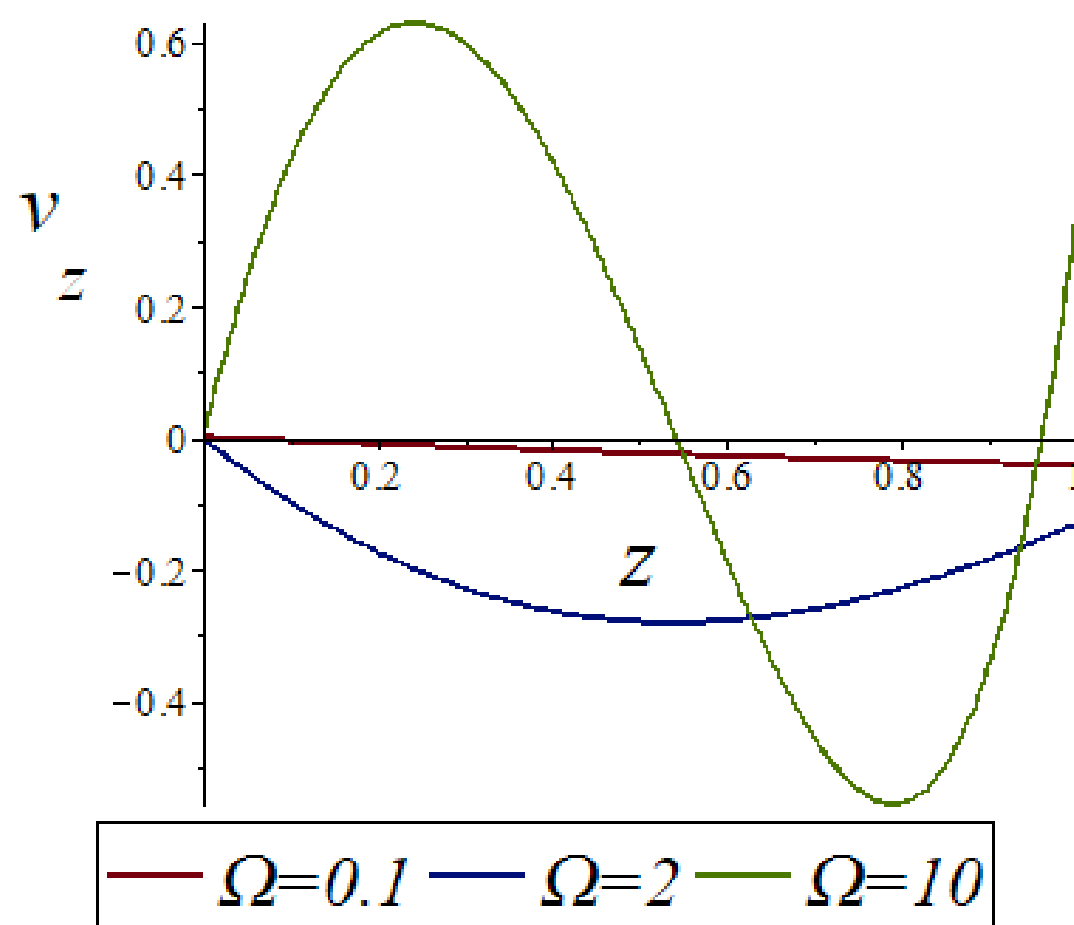
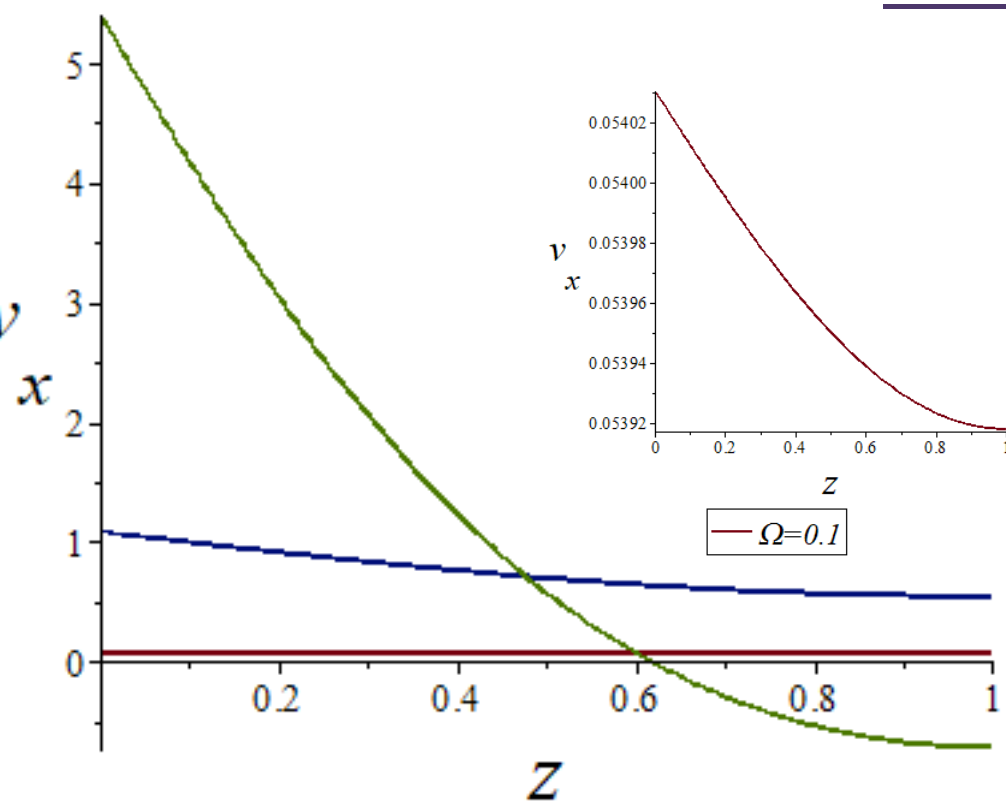
$$\begin{aligned} \text{r.y.} \\ z=0: \\ v_z = v_y = 0, v_x = A\Omega \cos(\Omega t) \end{aligned}$$

$$\begin{aligned} z=h: \\ \frac{\partial h}{\partial t} + \mathbf{v} \nabla h = v_z, \quad \frac{\partial T}{\partial z} = 1, \\ (p + \varphi - \sigma K) \mathbf{n} = \bar{\eta} \mathbf{n}, \\ \tau \bar{\eta} \mathbf{n} = -\frac{\partial \sigma}{\partial T} \tau \nabla T, \quad \mathbf{n} \nabla T = -\alpha T \end{aligned}$$

Dimensionless variables

$$\begin{aligned} [H] = h_0, [t] = \frac{h_0^2}{\nu}, [v] = \frac{\nu}{h_0}, \\ [p] = \frac{\rho \nu^2}{h_0^2}, \left[\frac{\partial T}{\partial z} \right] = \frac{T_0}{h_0} \end{aligned}$$

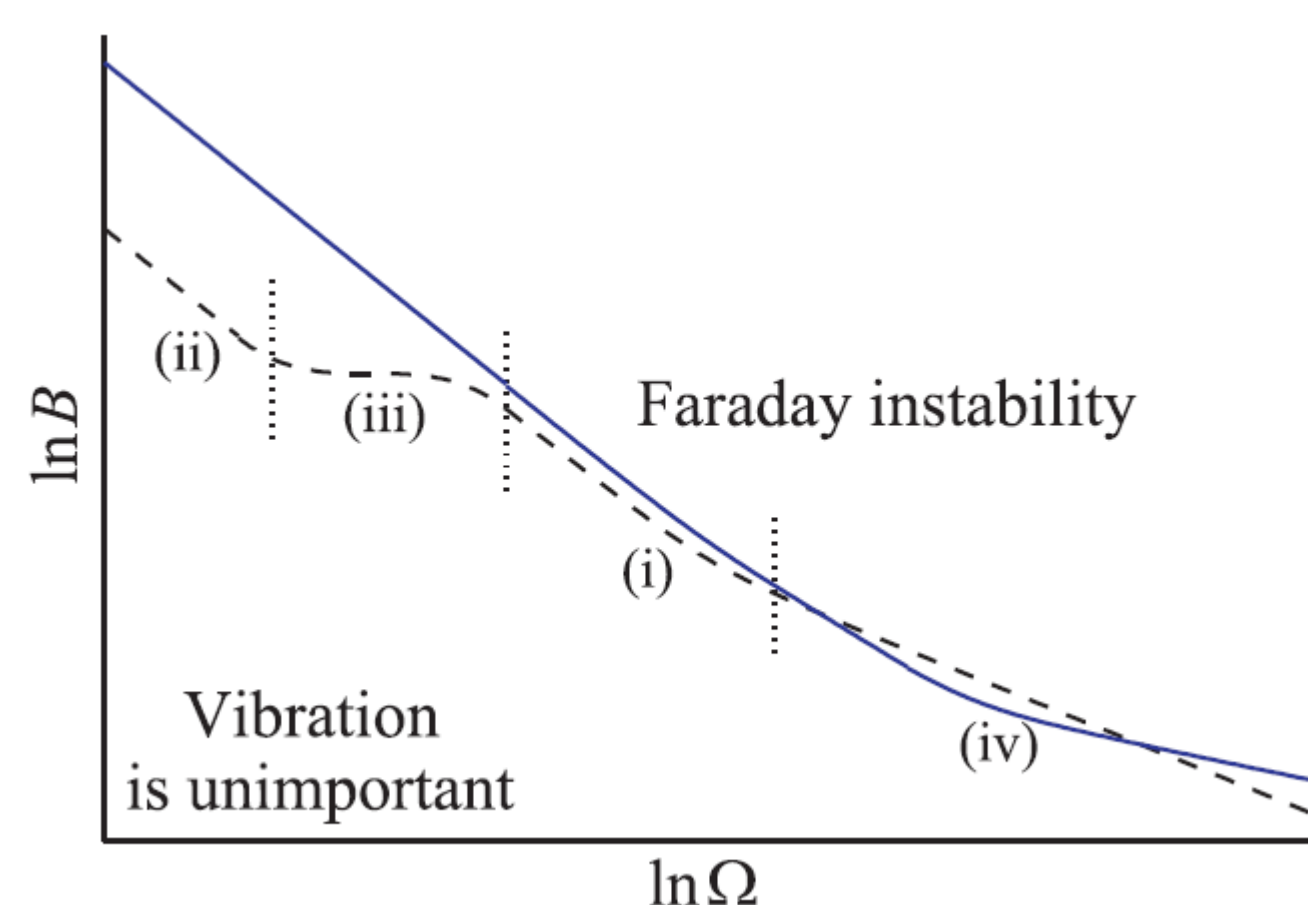
Pulsatile flow



$$\tilde{p} = \tilde{v}_y = 0.$$

$$\begin{aligned} \tilde{U} &= A\Omega \operatorname{Re} [v_x(X, Z) e^{i\Omega t}], \\ \tilde{W} &= A\Omega \operatorname{Re} [v_z(X, Z) e^{i\Omega t}], \\ \tilde{h} &= A \operatorname{Re} [H(X) e^{i\Omega t}], \\ \tilde{T} &= A\Omega \operatorname{Re} [T(X, Z) e^{i\Omega t}] \end{aligned}$$

Vertical vibrations



- (i) Average approximation $\Omega \sim O(1), b \sim O(\varepsilon^{-1})$,
- (ii) Ultralow frequency $\Omega \sim O(\varepsilon^2), b\Omega^2 \sim O(1)$,
- (iii) Low frequency $\Omega \sim O(\varepsilon^\beta), b \sim O(\varepsilon^{-1-3\beta/2}), 0 \leq \beta \leq 2$
- (iv) High frequency

U. Thiele, J. M. Vega, and E. Knobloch, J. Fluid Mech. 546, 61 (2006).

Method of solution

Average and pulsatile flows. Scalenig

Медленное время: $t = \varepsilon^2 t, \varepsilon = \frac{H}{L} \ll 1$.

Быстрое пульсационное время: $\tau = \Omega t$.

$$Z = z, X = \varepsilon x, Y = \varepsilon y.$$

$$\mathbf{v} = \langle \mathbf{v} \rangle + \tilde{\mathbf{v}}, v_z = \varepsilon^2 \langle v_z \rangle + \varepsilon \tilde{v}_z$$

$\langle \mathbf{v} \rangle$ - осредненная, $\tilde{\mathbf{v}}$ - пульсационная скорость

$$v_x = \varepsilon \langle v_x \rangle + \tilde{v}_x, v_y = \varepsilon \langle v_y \rangle + \tilde{v}_y,$$

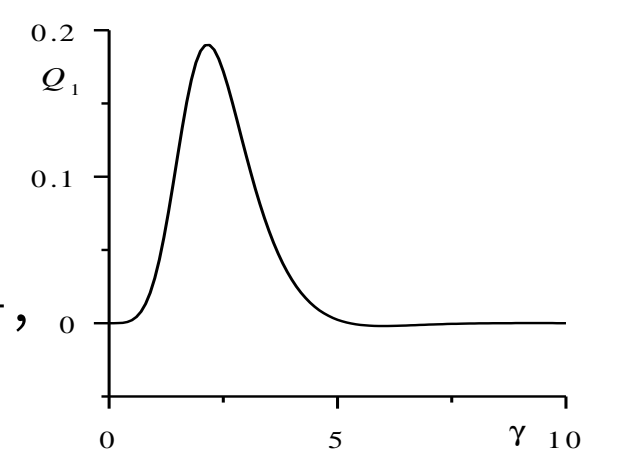
$$p = \langle p \rangle + \frac{1}{\varepsilon} \tilde{p}, h = \varepsilon \langle h \rangle + \tilde{h}, T = \langle T \rangle + \tilde{T}$$

Amplitude equation (non-isotherm case)

$$h_t = \frac{1}{3} \nabla \cdot (h^3 \nabla \Pi) - \frac{1}{2} A^2 \Omega^2 (Q_1 h^2 h_x)_x$$

$$\Pi = -\varphi(h) + Gh - C\Delta h, \quad \gamma = h\sqrt{2\Omega},$$

$$Q_1 = 3 \frac{2 \sin \gamma \operatorname{sh} \gamma - \gamma (\cos \gamma \operatorname{sh} \gamma + \operatorname{ch} \gamma \sin \gamma)}{\gamma^2 (\operatorname{ch} \gamma + \cos \gamma)^2}$$



Small perturbations

$$h = 1 + \xi(X, Y, T), V = \frac{A^2 \Omega^2}{2\alpha}, \gamma_0 = \sqrt{2\Omega}$$

$$\xi_T = \Delta [(G_0 - 1)\xi - \Delta \xi] - VQ_1(\gamma_0) \xi_{XX},$$

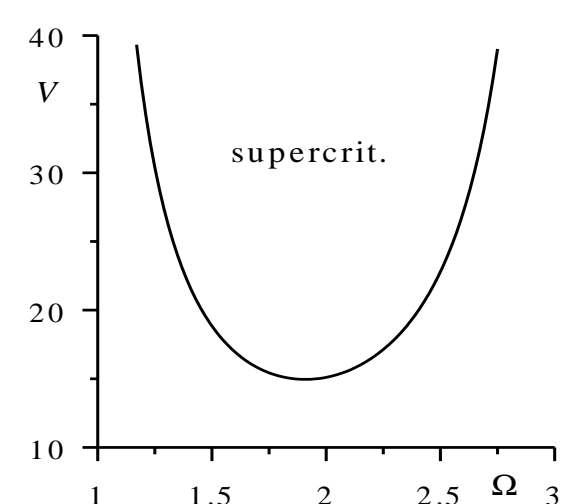
$$\xi = \hat{\xi} \exp(-\lambda T + ik_x X + ik_y Y),$$

$$\lambda = k^2 [G_0 - 1 + k^2] - VQ_1(\gamma_0) k_x^2.$$

Weakly nonlinear analysis

Стационарные решения малой амплитуды и малое отличие волнового числа от критического

$$h = 1 + \delta h_1 + \delta^2 h_2 + \dots, \quad \tilde{k} = \tilde{k}_c + \delta^2 \tilde{k}_2 + \dots$$



Unstable stationary solution

