ЛИПЕЦКИЙ ГОСУДАРСТВЕННЫЙ ТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ

Построение несимметричного термоупругого состояния анизотропного цилиндрического тела

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Анизотропые материалы

Rocks

Wood

Polymers

Composites

Polycrystalline metals

Construction Materials

Formulation of the problem

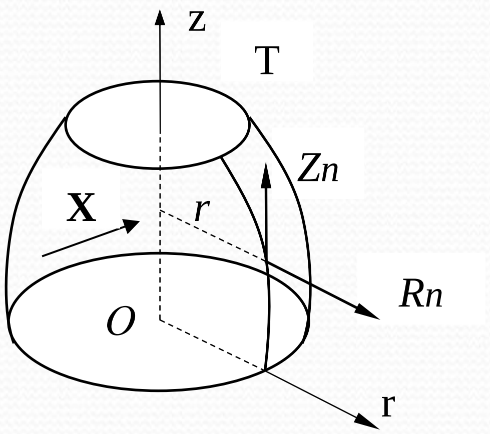


Fig. 1. A transversely isotropic body of revolution

ξ^0 - temperature elastic state;

ξ^X - elastic state from the action of mass forces;

ξ - elastic state due to surface forces;

$$\Omega = \xi^0 + \xi^X + \xi \quad \text{- total state.} \tag{1}$$

Elastic state - a set of components of the displacement vector, stress tensor, strain tensor and mass force vector.

Mathematical model

Boundary state method

$\xi = \{u_i, \varepsilon_{ij}, \sigma_{ij}\} \in \Xi$ - internal state;

$\gamma = \{u_i, p_i\} \in \Gamma$ - boundary state;

$$a_1\xi^{(1)} + a_2\xi^{(2)} \leftrightarrow a_1\gamma^{(1)} + a_2\gamma^{(2)}, \quad (\xi^{(1)}, \xi^{(2)})_\Xi = (\gamma^{(1)}, \gamma^{(2)})_\Gamma$$ - isomorphism.

The solution is the Fourier series:

$$\xi = \sum_l c_l \xi^{(l)}. \tag{2}$$

Expanded view:

$$u_i = \sum_{l=0}^{\infty} \tilde{n}_k u_i^{(l)}; \ \varepsilon_{ij} = \sum_{l=0}^{\infty} \tilde{n}_k \varepsilon_{ij}^{(l)}; \ \sigma_{ij} = \sum_{l=0}^{\infty} \tilde{n}_k \sigma_{ij}^{(l)}; \ X_i = \sum_{l=0}^{\infty} c_k X_i^{(l)}. \tag{3}$$

The solution of the boundary value problem

The general solution to the problem of plane deformation of a transversely isotropic medium:

$$\sigma_z{}^{pl} = -\mathrm{Re}[\gamma_1^2 \varphi_1^{'}(\varsigma_1) + \gamma_2^2 \varphi_2^{'}(\varsigma_2)]; \quad \sigma_y{}^{pl} = \mathrm{Re}[\varphi_1^{'}(\varsigma_1) + \varphi_2^{'}(\varsigma_2)];$$

$$\sigma_{zy}{}^{pl} = -\mathrm{Re}[\gamma_1 \varphi_1^{'}(\varsigma_1) + \gamma_2 \varphi_2^{'}(\varsigma_2)]; \quad \sigma_\eta{}^{pl} = \nu_r \sigma_y^{pl} + \nu_z \frac{E_r}{E_z} \sigma_z^{pl}; \quad \tau_{z\theta} = 0; \quad \tau_{r\theta} = 0; \qquad (4)$$

Transition to an axisymmetric spatial state:

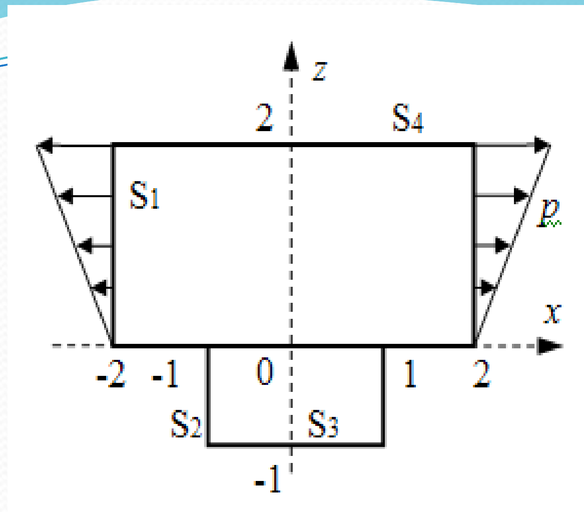
$$\sigma_z = \frac{1}{\pi} \int_{-r}^{r} \frac{\sigma_z^{pl}}{\sqrt{r^2 - y^2}} dy; \quad \sigma_{zr} = \frac{1}{\pi} \int_{-r}^{r} \frac{\sigma_{zy}^{pl}}{r\sqrt{r^2 - y^2}} dy; \quad \sigma_r - \sigma_\theta = \frac{1}{\pi} \int_{-r}^{r} \frac{(\sigma_y^{pl} - \sigma_\eta^{pl})(2y^2 - r^2)}{r^2 \sqrt{r^2 - y^2}} dy; \quad \sigma_{z\theta} = \sigma_{r\theta};$$

$$\sigma_r + \sigma_\theta = \frac{1}{\pi} \int_{-r}^{r} \frac{(\sigma_y^{pl} + \sigma_\eta^{pl})}{\sqrt{r^2 - y^2}} dy; \quad u = \frac{1}{\pi} \int_{-r}^{r} \frac{u_y^{pl}}{r\sqrt{r^2 - y^2}} dy; \quad w = \frac{1}{\pi} \int_{-r}^{r} \frac{u_z^{pl}}{r\sqrt{r^2 - y^2}} dy; \quad v = 0. \qquad (5)$$

$(\xi_1, \xi_2) = \int_V \varepsilon_{ij}^1 \sigma_{ij}^2 dV$ - scalar product in the space of internal states; (6)

$(\gamma_1, \gamma_2) = \int_S p_i^1 u_i^2 dS$ - scalar product in the space of boundary states; (7)

$c_k = \int_S (p_r u^k + p_z w^k) dS$ - Fourier coefficients. (8)



$p_r = z \cdot 10^2, p_z = 0, (r = 2,\ 0 \le z \le 2);$

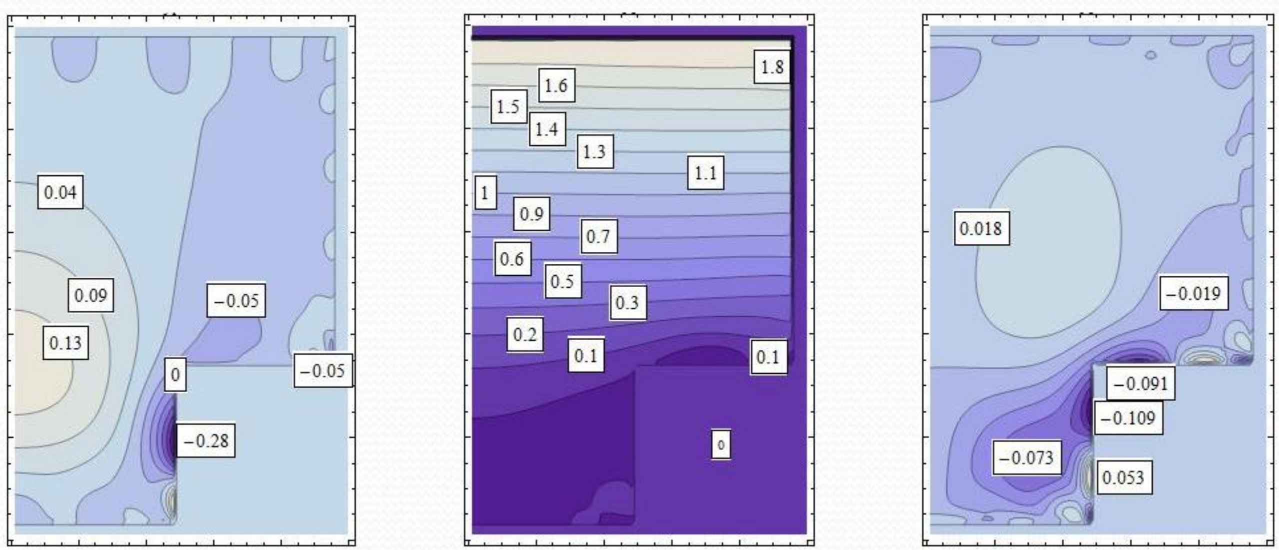
$p_r = 0, p_z = 0, (r = 1,\ -1 \le z \le 0);$

$p_r = 0, p_z = 0, (z = -1,\ 0 \le r \le 1);$

$p_r = 0, p_z = 0, (z = 2,\ 0 \le r \le 2);$

$\grave{O} = z^2;\ \mathbf{X} = 10 \cdot \{r^2, z^2 + r\}.$

Fig. 2. Meridian section of a body of revolution



a b v

Fig. 3. Isolines of stress tensor components: a - σ_{zz} ; b - σ_{rr}; v - σ_{zr}

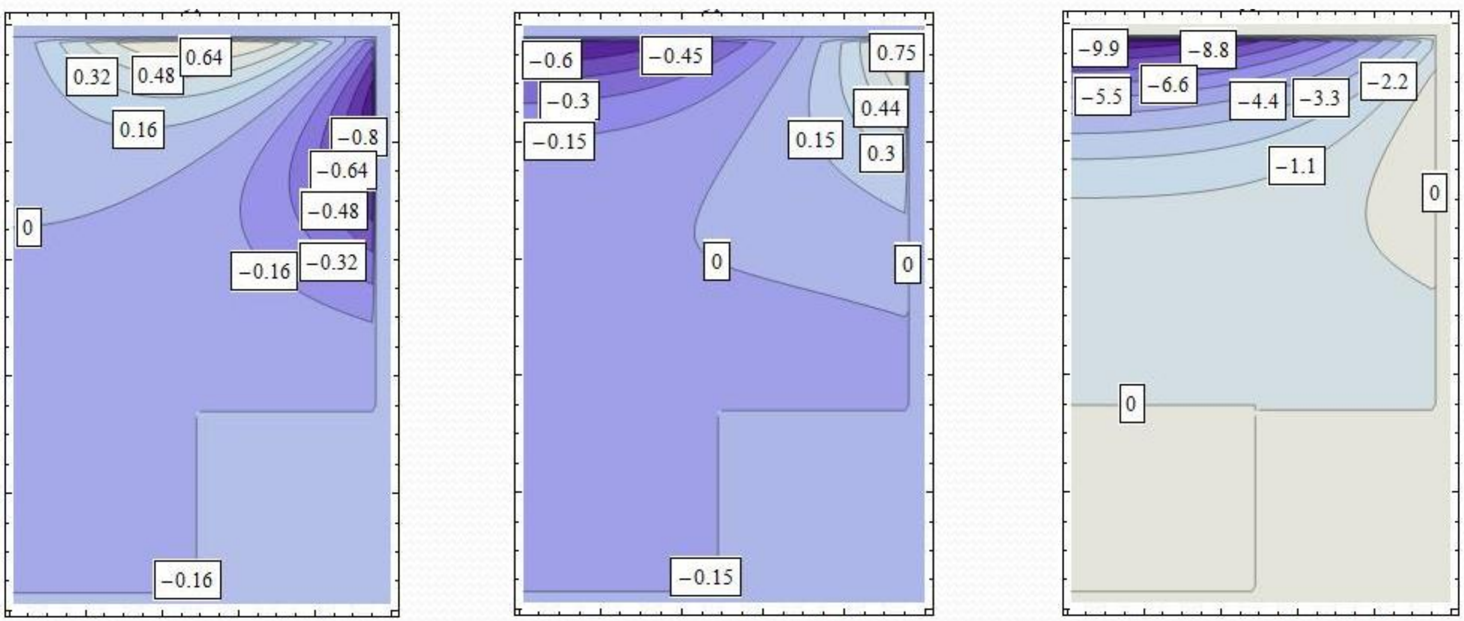
The solution of the problem from the action of mass forces

Fundamental polynomial system:

$$\mathbf{u}_{pl}^{X}=\left\{\left\{y^{\alpha}z^{\beta},0,\right\},\left\{0,y^{\alpha}z^{\beta}\right\}\right\}. \tag{9}$$

Scalar product and Fourier coefficients:

$$\left(\mathbf{X}^{(1)},\mathbf{X}^{(2)}\right)=\int_V \mathbf{X}^{(1)}\cdot\mathbf{X}^{(2)}dV;\;\; \mathbf{X}^{(k)}=\{R^{(k)}(r,z),Z^{(k)}(r,z)\};\;\; c_k^X=\left(\mathbf{X},\mathbf{X}_{\text{ïðò}}^{(k)}\right). \tag{10}$$



a b v

Fig. 4. Isolines : a - vector u ; b – vector w ; v - σ_{zz}

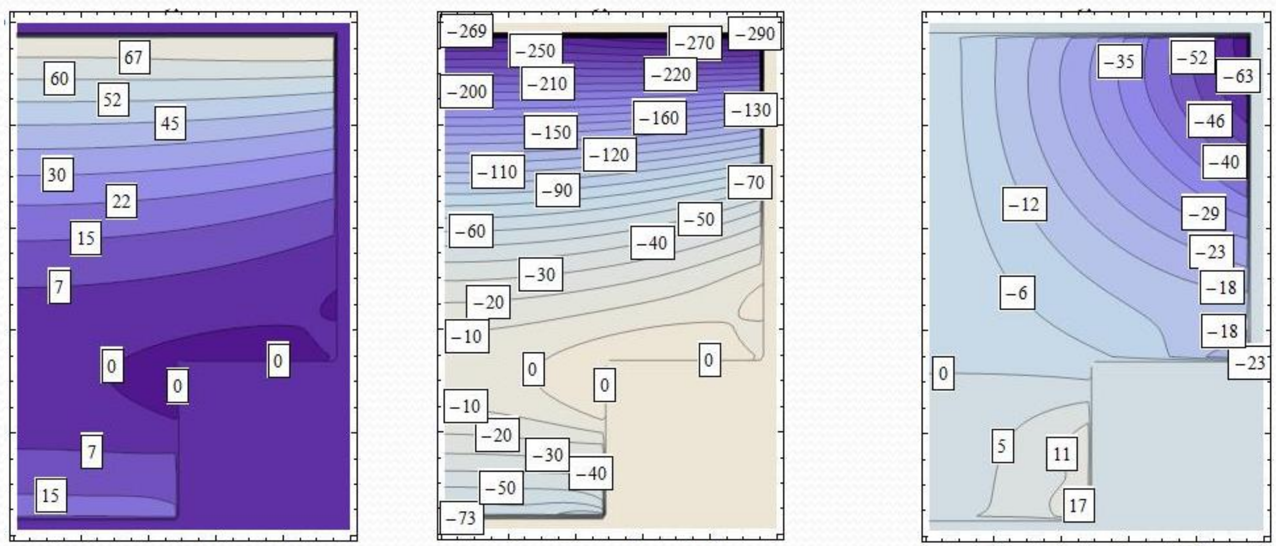
Solving the thermoelastic problem

General solution for flat temperature deformation:

$$T_0^{pl} = \frac{g_0}{E_z}\mathrm{Re}[\varphi_0'(\varsigma_0)]; \quad \varsigma_0 = z/\gamma_0 + iy; \quad u_z^{0\,pl} = \mathrm{Re}[p_0\varphi_0(\varsigma_0)]; \quad u_y^{0\,pl} = \mathrm{Re}[iq_0\varphi_0(\varsigma_0)]; \quad u_\eta^{0\,pl} = 0;$$
$$\sigma_z^{0\,pl} = -\mathrm{Re}[\gamma_0^2\varphi_0'(\varsigma_0)]; \quad \sigma_y^{0\,pl} = \mathrm{Re}[\varphi_0'(\varsigma_0)]; \quad \sigma_{zy}^{0\,pl} = -\mathrm{Re}[\gamma_0\varphi_0'(\varsigma_0)]; \quad \sigma_\eta^{0\,pl} = \mathrm{Re}[(1-\varepsilon_0)\varphi_0'(\varsigma_0)]; \qquad (11)$$
$$\tau_{z\theta} = 0; \quad \tau_{r\theta} = 0.$$

Scalar product and Fourier coefficients:

$$(\xi_1^0, \xi_2^0) = \int_V T_1^0 T_2^0\, dV; \quad c_k^0 = \int_V T\, T_k^0\, dV. \qquad (12)$$



a b v

Fig. 5. Isolines of stress tensor components: a - σ_{zz} ; b - σ_{rr}; v - σ_{zr}

Resulting state

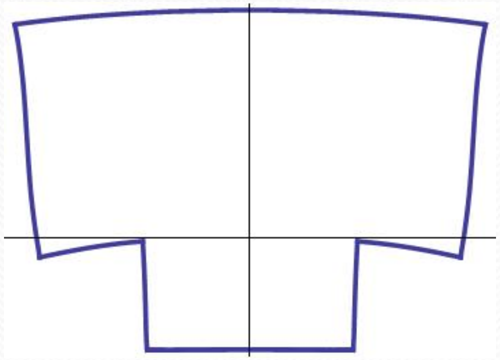
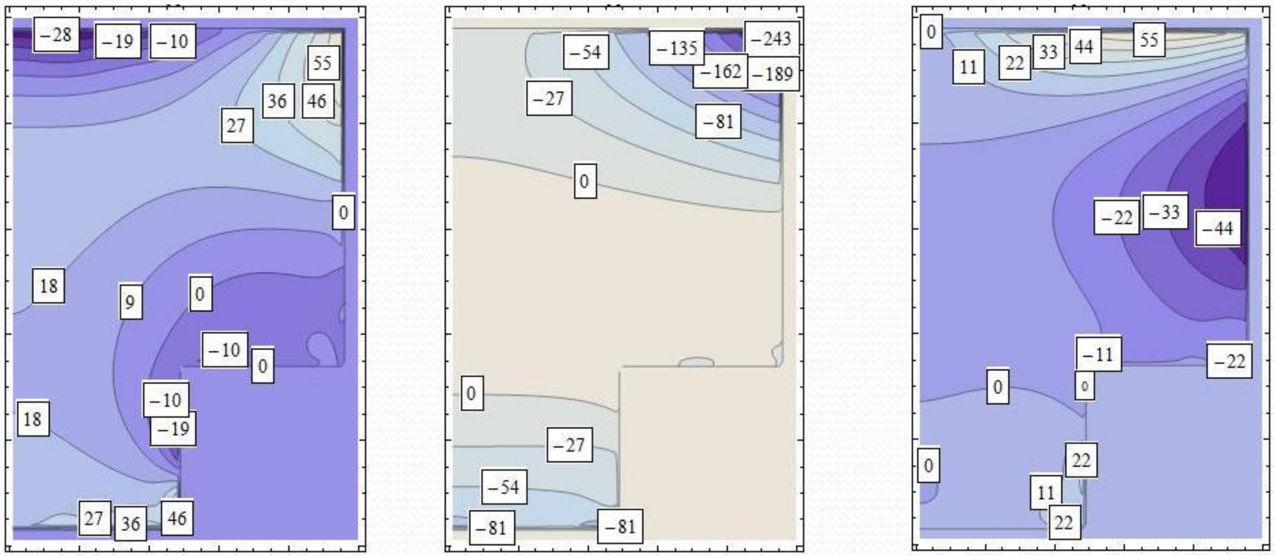


Fig. 6. The contour of the deformed state



a b v

Fig. 7. Isolines of stress tensor components: a - σ_{zz} ; b - σ_{rr} ; v - σ_{zr}

Thanks for attention !